Possible Mechanisms of Clear-Air Turbulence in Strongly Anticyclonic Flows

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ABSTRACT

The forecasting of clear-air turbulence (CAT) continues to be a challenging problem despite progress made in the understanding of vertical shear (Kelvin–Helmholtz) instabilities. The possible connections between horizontal anticyclonic flows and CAT are addressed. Analytical expressions are derived to show that current CAT diagnostics do not correctly account for the dynamics of strongly anticyclonic situations. In gradient-balanced anticyclonic flows, nonfrontogenetical enhancement of vertical shear may lead to CAT. A review of observations, theory, and modeling is presented to support the claim that strong anticyclonic relative vorticity can also lead to CAT through the generation of gravity wave activity by geostrophic adjustment and inertial instability. CAT diagnostics are then discussed in light of these claims. Observational work is in progress to investigate the possibility of inertial instability-triggered CAT.

1. Introduction

Clear-air turbulence (CAT) is defined as the detection by aircraft of high-altitude inflight bumps in patchy regions devoid of significant cloudiness or nearby thunderstorm activity (Chambers 1955). The ''buffeting effect'' due to CAT is ''similar to that experienced in running a fast boat over an area of choppy water'' (Sowa 1966, unpublished manuscript[1]) and in the most severe instances can lead to aircraft damage and personal injuries. This phenomenon was first noted in the 1940s (e.g., Baughman 1946), and was widely recognized as an aircraft safety hazard in the 1960s when tens of millions of dollars in damage were attributed to CAT by the U.S. military and commercial aviation communities (Dutton and Panofsky 1970). Even in the 1990s, CAT is a primary cause or factor in several aviation mishaps annually (National Transportation Safety Board 1993, 66). Recent well-publicized events make obvious the fact that CAT remains a safety hazard to commercial aviation today (Phillips 1995).

Early seminal research on aviation turbulence linked CAT to the presence of vertical (and horizontal) wind shear (e.g., Sowa 1966, unpublished manuscript). The mystery of CAT was thought to be solved when its connection to vertical shear instabilities was discovered (Dutton and Panofsky 1970). Notable improvements have been made in mountain wave turbulence forecasting (e.g., Bacmeister et al. 1994). A quarter-century later, however, our understanding of CAT still does not permit explanations and forecasts of a sizable number of occurrences away from mountainous regions, particularly moderate and severe CAT events (Dutton 1980). McCann (1993), using December 1992–February 1993 aircraft reports and rawinsonde analyses, demonstrated that correlation coefficients linking CAT occurrence and operational CAT diagnostics are often less than ± 0.1 and never greater than ± 0.35. This result suggests that conventional CAT diagnostics may require reexamination.

Herein, attention is focused on mechanisms of CAT and its diagnosis in regions of strong anticyclonic flow, which have long been associated with some CAT occurrences (e.g., Colson 1969). In section 2, the role of frontogenetical deformation in causing CAT in strongly anticyclonic flow is considered. It is argued that the conventional linkage between frontogenesis, deformation, and CAT is not appropriate in strongly anticyclonic flows. In section 3, a nonfrontogenetical mechanism for the production of strong vertical shear in gradient-balanced anticyclonic flows is analyzed using simple dynamical arguments. In section 4, the possibility of CAT generation in unbalanced anticyclonic flows due to the mechanisms of geostrophic adjustment and inertial in-

[1] High altitude meteorology fundamentals; available from Northwest Airlines, Inc., 5101 Northwest Dr., St. Paul, MN 55111.

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stability is discussed. The linkage between CAT and these mechanisms is justified with reference to an array of published observational, theoretical, and modeling results. In section 5, conventional CAT diagnostics are examined in light of the foregoing discussion. Section 6 gives a brief summary and a prospectus for future work.

2. Conventional CAT mechanisms in anticyclonic flows

a. Background

Vertical shear of the horizontal wind is recognized as a primary cause of CAT. Surprisingly, studies have shown that large magnitudes of $\partial \mathbf{u}/\partial z$ by themselves are not the best indicator of CAT (Mancuso and Endlich 1966; Dutton 1980). Two well-known diagnostics of CAT (Keller 1990; Ellrod and Knapp 1992) therefore employ measures of horizontal as well as vertical shear, in keeping with the observation that CAT often occurs in regions where the flow changes markedly in all three spatial directions. Both of these indices link CAT with large values of horizontal deformation terms (or their squares), defined as

$$\mathrm{DST} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \tag{1}$$

where DST is the stretching deformation and u and v are the horizontal components of the wind in the x (zonal) and y (meridional) directions; and

$$\mathrm{DSH} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \tag{2}$$

where DSH is the shearing deformation (Saucier 1955, 355–356).

Flow deformation was first suggested as a producer of CAT by Mancuso and Endlich (1966). The original impetus for linking deformation and CAT seems to have been largely empirical (e.g., Roach 1970; Brown 1973). Mancuso and Endlich and most subsequent researchers justified the use of deformation in CAT studies in the following way, after Petterssen (1956, chapter 11): horizontal deformation can increase horizontal gradients of temperature, which by the thermal wind relationship leads to enhanced vertical gradients of the horizontal wind, which lowers the Richardson number (if the static stability is held constant) and increases the probability of in situ Kelvin–Helmholtz instabilities. In Ellrod and Knapp's (1992) turbulence indices (TI), this can be expressed mathematically through Petterssen's frontogenetic intensity, in which frontogenesis is linearly proportional to deformation. Alternatively, Keller's (1990) ''SCATR index'' linearly relates the nonturbulent component of the tendency of the Richardson number to DST and DSH. Dutton (1980), however, provided a different interpretation of the deformation–CAT connection: regions of strong vertical shear are usually tilted, and so a component of the vertical shear is projected onto the horizontal.

The preceding causal argument linking flow deformation to CAT via frontogenesis has been repeated often enough to be accepted without question.[2] However, it is interesting to note that in the paper that first suggested this linkage, Mancuso and Endlich (1966) initially considered an alternate mechanism for CAT generation: the generation of mesoscale waves by regions of large deformation. Only after the authors noted the then-current lack of theoretical support for mesoscale wave generation did they present the frontogenetical argument outlined above, which they felt was ''not . . . completely satisfactory.'' Nevertheless, this argument has since become the paradigm for the deformation–CAT relationship. Below, we examine more closely the relationship between deformation, vertical shear, and anticyclonic flows.

b. Dynamical interpretation

We now consider deformation and frontogenesis in rotating flows. Defining the total flow deformation DEF as

$$\mathrm{DEF}^2 = \mathrm{DST}^2 + \mathrm{DSH}^2, \tag{3}$$

the horizontal deformation terms DST and DSH can be reexpressed in terms of more familiar dynamical quantities;

$$\mathrm{DEF}^2 = \zeta^2 + D^2 - 4J(u, v), \tag{4}$$

where ζ is the relative vorticity $\partial v/\partial x - \partial u/\partial y$, D is the horizontal divergence $\partial u/\partial x + \partial v/\partial y$, and $J(u, v)$ is the Jacobian operator $(\partial u/\partial x)(\partial v/\partial y) - (\partial v/\partial x)(\partial u/\partial y)$. No assumptions are involved in this simple algebraic rearrangement.

Furthermore, in exact gradient-balanced flow the Jacobian term can be reexpressed in terms of vorticity as $-4\zeta_{\mathrm{shear}}\zeta_{\mathrm{curv}}$. Here, $\zeta_{\mathrm{shear}} = -\partial V/\partial n$ and $\zeta_{\mathrm{curv}} = V/R$ are the shear and curvature components of the relative vorticity, in which V is the flow speed, R is the radius of curvature of streamlines, and n is the direction of the normal vector, which points to the left of the flow. In addition, $\zeta_{\mathrm{shear}} + \zeta_{\mathrm{curv}} = \zeta$ (Holton 1992, 95–96).

For strong cyclonic flows, ζ is larger than the Coriolis parameter $f = 2\Omega \sin\phi$, which in these cases is much larger than the divergence D. Consequently, (4) reduces to

$$\mathrm{DEF}^2 \approx k\zeta^2, \tag{5}$$

where the coefficient k is

[2] A reviewer observes, however, that this argument ignores the modern frontogenesis theory of Hoskins and Bretherton (1972), et seq.

$$k = \begin{cases} > 1, & \zeta_{\rm shear}/\zeta_{\rm curv} < 0 & \text{e.g., outer Rankine vortex} \\ 1, & \zeta_{\rm shear} = 0 \quad \text{or} \quad \zeta_{\rm curv} = 0 & \text{e.g., pure shear flow} \\ < 1, & \zeta_{\rm shear}/\zeta_{\rm curv} > 0 \quad \text{and} \quad \zeta_{\rm shear} \neq \zeta_{\rm curv} & \text{e.g., front, cutoff low} \\ 0, & \zeta_{\rm shear} = \zeta_{\rm curv} \neq 0 & \text{e.g., pure circular flow.} \end{cases}$$

This relationship between DEF^2 and ζ^2 agrees with the qualitative analysis by Saucier (1953; see in particular Fig. 8c). Further details on the relative signs and magnitudes of $\zeta_{\rm shear}$ and $\zeta_{\rm curv}$ in cyclonic flow can be found in Bell and Keyser (1993). From (5), it is seen that large values of deformation will coincide with noncircular regions of large cyclonic relative vorticity, such as fronts. Hence, a linkage between deformation and frontogenesis is possible in cyclonic flow.

For strong anticyclonic flows, interpretation of deformation as frontogenesis is more problematic. For very strong anticyclonic horizontal shear and/or curvature where $\zeta \approx -f$, once again the square of the relative vorticity usually dominates the square of the divergence in (4). This was verified by Angell (1961, Table VII) using constant-pressure balloon data for the upper troposphere. Angell (1961, 207–208) also concluded that ''the horizontal deformation is intermediate in magnitude to the vertical component of vorticity and the horizontal divergence.'' This conclusion supports the derivation of (5) above, which retains ζ and a (usually subtractive) contribution from the Jacobian term while ignoring D. *Therefore, (5) should hold for strong anticyclonic flows as well as cyclonic flows.*

Based on (5), the deformation may be large or negligibly small in anticyclonic flows, depending on the relative magnitudes of the shear and curvature. For example, a strongly anticyclonically sheared straight jet ($k = 1$) will yield large values of DEF^2, but a tightly curved ridge with strong anticyclonic shear ($k \approx 0$) will not. Angell (1961, his Fig. 23), using constant-pressure balloon trajectories to estimate ζ, D, and DEF, found that in a strongly anticyclonically curved and sheared upper-tropospheric ridge $\mathrm{DEF}^2 = 0.2\zeta^2$. This matches expectations based on (5), since for Angell's case k would be expected to be $\ll 1$. Therefore, there is both theoretical and observational justification for the idea that strongly anticyclonic flows can be characterized by either weak or strong deformation.

In the case of straight-line, strongly anticyclonically sheared flow ($k = 1$), are the large values of deformation predicted by (5) interpretable as frontogenesis? Petterssen's frontogenetic intensity equation is purely kinematic, not dynamical, and thus does not recognize the well-known asymmetry in the allowed strengths of highs and lows (Haltiner and Martin 1957, 191). In the more sophisticated Sawyer–Eliassen theory, the presence of negative absolute vorticity can lead to nonzero solutions in the absence of frontogenesis (Sawyer 1956). From an observational perspective, Saucier (1955, 367, 372–373) permits the existence of frontogenesis along a ridge, but only in a local trough or a weakness in the ridge. Thus, the argument for the connection between deformation, frontogenesis, and CAT would seem to be on very shaky ground in the vicinity of centers of large negative relative vorticity.

Furthermore, in regions of strong anticyclonic shear *and* curvature ($k \ll 1$), from (5) it is seen that DEF^2 will be quite small. This is illustrated for geostrophic deformations in Petterssen (1953, his Figs. 2, 8). Thus, if such regions are generators of CAT, no appeal can be made to the traditional frontogenetical-deformation arguments for the creation of turbulence through unstable vertical shears.

3. Ageostrophic vertical shear in gradient-balanced flows

The enhancement of vertical shears—which is the desideratum of any conventional argument for CAT production—can be linked to anticyclonic flow without explicit reference to deformation or frontogenesis. By differentiating the gradient wind in the vertical, one obtains the following (adapted from Newton and Palmén 1963 and Reed and Hardy 1972):

$$\frac{\partial V_{\rm gr}}{\partial z} = \left(f\frac{\partial V_g}{\partial z} + \frac{V_{\rm gr}^2}{R^2}\frac{\partial R}{\partial z}\right)\left(f + \frac{2V_{\rm gr}}{R}\right)^{-1}, \qquad (6)$$

where V_g is the geostrophic wind speed and $V_{\rm gr}$ is the gradient wind speed. Reed and Hardy (1972, their Table 2) found that for a case of strong anticyclonic flow associated with CAT but not connected to any frontal system, the *vertical shear* was markedly supergeostrophic.

This finding can be made more physically interpretable by deriving an expression for the ageostrophic vertical shear in gradient flow. By splitting $V_{\rm gr}$ in (6) into geostrophic (V_g) and ageostrophic ($V_{\rm ag}$) components, the result is

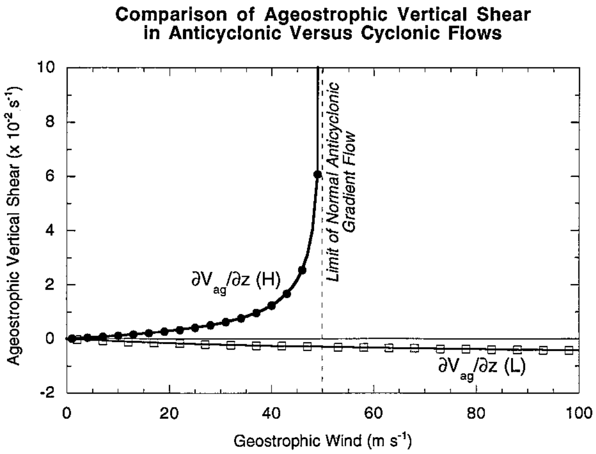

FIG. 1. Comparison of ageostrophic vertical shear for anticyclonic and cyclonic conditions as a function of the geostrophic wind, based on Eq. (7). Flow assumptions: latitude 43°N; radius of curvature $R = \pm 2 \times 10^6$ m, unchanging with altitude; geostrophic vertical shear $= +1$ m s^{-1} (100 m)$^{-1}$. Solid circles indicate the anticyclonic ageostrophic vertical shear (labeled with "H"; $R = -2 \times 10^6$ m); squares denote the cyclonic ageostrophic vertical shear (labeled with "L"; $R = +2 \times 10^6$ m). Note that for the assumed flow parameters, $V_g \geq 50$ m s^{-1} is not permitted in normal gradient flow.

$$\frac{\partial V_{ag}}{\partial z} = \underbrace{-2\frac{V_{gr}}{R}\frac{\partial V_g}{\partial z}\left(f + \frac{2V_{gr}}{R}\right)^{-1}}_{\text{horizontal curvature}} + \underbrace{\frac{V_{gr}^2}{R^2}\frac{\partial R}{\partial z}\left(f + \frac{2V_{gr}}{R}\right)^{-1}}_{\text{gradient baroclinicity}}. \qquad (7)$$

The first right-hand-side term in (7) describes the ageostrophic component of the vertical shear whose sign and magnitude is dependent on the sign and magnitude of the horizontal radius of curvature R. (It is also dependent on the sign and magnitude of the geostrophic vertical shear; in the following discussion we will fix $\partial V_g/\partial z$ and vary R.) The second right-hand-side term in (7) describes the ageostrophic component whose sign is dependent on the sign of $\partial R/\partial z$, that is, the baroclinic nature of the gradient flow. When $R \rightarrow \infty$ both the "horizontal curvature" and "gradient baroclinicity" terms disappear and $\partial V_{ag}/\partial z = 0$, as would be expected for straight-line geostrophic flow. When $\partial R/\partial z = 0$, the gradient baroclinicity term disappears and all ageostrophic effects are due solely to the flow curvature; we now consider this simplified case in Fig. 1.

Figure 1 graphically depicts (7) for a midlatitude situation in which $\partial V_g/\partial z = +1 \times 10^{-2}$ s^{-1} and the horizontal curvature is constant with height. Two results are obvious from (7) and this figure—the first dealing with the *sign* of the ageostrophic wind shear, the second pertaining to the *magnitude* of the ageostrophic wind shear. First, $\partial V_{ag}/\partial z$ in an anticyclone is the same sign as $\partial V_g/\partial z$, but $\partial V_{ag}/\partial z$ opposes $\partial V_g/\partial z$ in a cyclone. In other words, the total vertical shear—geostrophic plus ageostrophic—is greater in a ridge than a trough for the same absolute values of pressure gradient and flow curvature. This agrees with Reed and Hardy's (1972) findings and also with comments by Sanders (1986, 1857–1858).

Second, the magnitude of the ageostrophic vertical shear is larger in the anticyclonic case than in the cyclonic case, with this asymmetry becoming most pronounced for strongly anticyclonic flows. For example, in Fig. 1 for $V_g = 22$ m s^{-1} the ratio of anticyclonic $|\partial V_{ag}/\partial z|$ versus cyclonic $|\partial V_{ag}/\partial z|$ is 2.02. However, at $V_g = 42$ m s^{-1} (closer to the limit of gradient-balanced anticyclonic flow for the assumptions in Fig. 1) this ratio is 5.71. Also note that although $|\partial V_{ag}/\partial z|$ for cyclonic flow is never more than about 40% of the geostrophic value used in Fig. 1, the anticyclonic $\partial V_{ag}/\partial z$ easily exceeds $\partial V_g/\partial z$ for strongly anticyclonic cases, that is, as V_g approaches 50 m s^{-1}.[3]

Both of these effects contribute to the significant enhancement of vertical shears in an anticyclonic flow pattern, over and above what would be expected from geostrophic vertical shear calculations. The conclusion: *strongly anticyclonic flows are regions in which very strong vertical shears may exist due to ageostrophic dynamics.* As is clear from Fig. 1, this conclusion is valid independent of any vertical changes in flow curvature; this point seems to have been overlooked in recent discussions [Bluestein 1993, 388; also cf. his Eq. (2.7.9) to Eq. (7)]. In Reed and Hardy's case the vertical derivative of R was *not* significant despite strong anticyclonicity. Newton and Palmén (1963) applied (6) to two regions, and found that vertical changes in curvature were negligible in one case and highly significant in the other.

An important aspect of the above discussion is that the markedly supergeostrophic vertical shears in anticyclonic flows are unrelated to frontogenetical arguments. Reed and Hardy (1972, 548) speculated that "regions of large [vertical] shear not directly connected with frontal structure may exist in the vicinity of anticyclonically curved jet streams." And so anticyclonic flow curvature, not directly connected to frontogenesis, may play a role in leading to vertical shear instabilities.

Despite the usefulness of this explanation, it must be emphasized that this mechanism holds only for flow in

[3] In the strongest balanced anticyclonic flows, there is a significant restriction on the permissible values of the geostrophic vertical shear (Sanders 1986); this may mute somewhat the importance of the steepest portion of the anticyclonic shear curve in Fig. 1, since the total shear in such cases may not be enough to lead to very small values of Ri. However, Reed and Hardy's (1972) case study contains anticyclonically curved regions in which both the observed vertical shear and the calculated ageostrophic vertical shear were as much as three times larger than the geostrophic shear—*and* the Richardson number was less than unity.

steady gradient balance. Gradient balance is a poor assumption for strongly anticyclonic flow: for example, such flows violate the ellipticity criterion, which defines the solvability of the nonlinear balance equation (a generalization of gradient balance) (Daley 1991). Violations of normal gradient wind balance can occur when jet streaks enter upper-tropospheric ridges (e.g., Uccellini et al. 1984; Moore and Abeling 1988), and also in cases of negative absolute vorticity or abnormal gradient flow (Alaka 1961). In steady abnormal flow, $2V_{gr}/R < -f$, and so from (6) the sign of $\partial V_{gr}/\partial z$ would usually be opposite to that of $\partial V_g/\partial z$. The presence of abnormal flow *or* negative absolute vorticity is a necessary condition for inertial instability, which does not satisfy the assumptions of steady flow. Although the meteorological relevance of abnormal gradient flow has been in doubt for decades (e.g., Mogil and Holle 1972; Leary 1974), the existence of inertial instability is on much firmer ground, as we shall examine in the next section.

In summary, the conventional mechanisms for CAT generation described in section 2 do not seem to govern centers of strong-to-extreme anticyclonic flow. Mechanisms linking deformation to frontogenesis may not be applicable in the strongest highs and ridges; furthermore, the foregoing dynamical analysis reveals that it is possible to create strong vertical shears in gradient-balanced anticyclonic flow without the presence of frontogenesis. Since anticyclonic regions are known to exhibit CAT (e.g., Chambers 1955; Sorenson 1964; Colson 1969), other CAT-producing mechanisms may also exist which are not accounted for in standard theory. Below, we discuss *nonequilibrium* anticyclonic states with respect to mechanisms of CAT generation.

4. Adjustment mechanisms and CAT generation in strongly anticyclonic flows

In section 2, it was pointed out that frontogenetical deformation explanations cannot explain the presence of CAT in strongly anticyclonically sheared and curved flows. In this section, we explore the possibility that these regions do in fact generate CAT, through the mechanisms of geostrophic adjustment and inertial instability.

As noted above, there is an asymmetry in the strengths and sizes of highs and lows—lows and troughs may be as small and intense as can be forced, but highs and ridges cannot be smaller or more intense than a certain limit (Holton 1992, 67–68). This asymmetry is imposed by the one-signedness of the earth's rotation, which in classical theory does not permit motions that have an absolute rotation in a sense opposite to the earth. Violations of this asymmetry are corrected by processes known as symmetric or inertial instability. For a straight geostrophic flow in thermal wind balance, the necessary criterion for this instability is (Stone 1966)

$$f[f(1 - \mathrm{Ri}^{-1}) + \zeta] < 0, \tag{8}$$

where $\mathrm{Ri} = N^2(\partial u/\partial z)^{-2}$ is the Richardson number, in which N is the Brunt–Väisälä frequency. When $\mathrm{Ri} < f(f + \zeta)^{-1}$, (8) describes symmetric instability, a slantwise mesoscale cellular circulation along sloping isentropic surfaces (Bluestein 1993, 342). When $\mathrm{Ri} \gg 1$, instances in which (8) is satisfied exhibit pure inertial instability, a horizontal mesoscale or large-scale cellular ''pancake'' circulation (Hitchman et al. 1987). It is clear from (8) that pure inertial instability occurs when the product of f and the absolute vorticity $f + \zeta$ is less than zero. Equivalently, in a statically stable atmosphere pure inertial instability is characterized by negative potential vorticity (Hoskins 1974). The discussion below focuses on inertial instability.

Inertial instability was first linked to CAT during the 1950s (e.g., Arakawa 1952; Schaefer and Hubert 1955). Chambers (1955) noted that perhaps no more than 10% of CAT occurrences were in the anticyclonic sector of jet streams. However, four of the six cases of very severe turbulence depicted in Chambers's paper occurred in these regions, where the absolute vorticity and inertial stability are typically very low. Sorenson (1964) suggested that the coincidence of CAT reports under an amplifying thermal ridge was due to the release of eddy energy by the centrifugal force term when the flow curvature reached a critical value. [This mechanism was also alluded to in a different context by Syōno (1948) and Miyakoda (1956).] Scorer (1969) postulated that inertial instability could lead to Kelvin–Helmholtz instabilities indirectly, by increasing vertical shears through cellular overturning. Mogil and Holle (1972) noted a correspondence between a moderate-to-severe CAT outbreak over Texas and a sharply curved ridge, which exhibited probable negative absolute vorticity and supercritical curvature. However, at that time the very existence of inertial instability was being questioned (e.g., Blumen and Washington 1969; Leary 1974); this debate, combined with the then-prevailing view that the CAT mystery was solved, appears to have inhibited theoretical research along these lines.

Sparks et al.'s (1977) observational investigation provides some of the clearest evidence for a linkage between strongly anticyclonic flow and CAT. Seven days of aircraft data over Europe were used, which included three strong ridging episodes. Sparks et al. (1977, 31) found that ''there is no evidence of a useful relationship between [deformation] and bumpiness but the relationship between [absolute vorticity] and bumpiness is more interesting.'' Although only 1% of aircraft reports came from regions that were predicted to have negative absolute vorticity, over 40% of these reports from classically inertially unstable regions detected at least slight turbulence—twice as high a percentage as for strongly cyclonic flow ($f + \zeta > 2.4 \times 10^{-4}$ s^{-1}). Above-background levels of moderate and severe turbulence were also noted for weakly inertially stable flows ($f + \zeta < 2 \times 10^{-5}$ s^{-1}, representing about 8% of the total number of pilot reports). However, the empirical CAT index derived by Sparks et al. did not employ absolute vorticity as a factor. A follow-up study by Dutton (1980) appears not to have captured either very strong cyclonic or anticyclonic cases and thus downplayed the relationship between absolute vorticity and CAT—lumping together all values of $\zeta + f$ less than $6 \times 10^{-5}\ \mathrm{s}^{-1}$.

Today, ''there seem to be good reasons for believing that inertial instability may frequently occur in the real atmosphere'' (Clark and Haynes 1994), although probably not as frequently as CAT. Pure inertial instability has been observed in the equatorial middle atmosphere (Hitchman et al. 1987) and modeled by several different research groups (e.g., O'Sullivan and Hitchman 1992; Sassi et al. 1993). Inertial instability has been inferred from observations of the midlatitude upper troposphere (Angell 1962; Ciesielski et al. 1989). Symmetric instability in the troposphere has been observed and modeled more extensively (e.g., Wolfsberg et al. 1986; Jones and Thorpe 1992), and large and persistent regions of negative moist potential vorticity have been noted in the upper-level wake regions of midlatitude squall lines in observations and models (e.g., Biggerstaff and Houze 1991; Zhang and Cho 1992, 1995; J. Martin 1995, personal communication).

Along with the renewed interest in inertial instability has come a better (but still rudimentary) understanding of how the approach to inertial instability can lead directly to gravity wave generation. As anticyclonic flows strengthen toward the limit of inertial stability, conventional balances—geostrophic, gradient, and nonlinear (Charney) balance—are violated and at some point the flow reacts to the loss of balance. This process is referred to as ''the spontaneous breakdown of balance and resulting gravity wave emission'' (McIntyre 1987, 280–282):

> Although spontaneous geostrophic adjustment is far from well understood, there is little doubt about its physical reality . . . there are also close cross-disciplinary analogues in aerodynamic sound generation . . . Both [meteorological studies] and aeroacoustic analogues indicate that the strength of the [gravity wave] emission process increases very steeply as local Rossby . . . numbers increase toward values of order unity.

By this argument, flows that approach or achieve inertial instability (which is characterized by Rossby numbers of order 1) should generate gravity waves. Tribbia (1981) noted this possibility with respect to nonelliptic regions, which are also strongly anticyclonic. A variety of modeling studies confirm this analogy. Van Tuyl and Young (1982) found that gravity–inertia modes increased markedly in amplitude as a jet was forced to approach the limit of inertial stability. Jones and Thorpe (1992) noted inertia–gravity wave emission from a region of symmetric instability. O'Sullivan (1993) simulated inertia–gravity wave generation from a large-scale region of inertial instability in the extratropical middle atmosphere. These waves were confined to regions equatorward of the inertially unstable zone. Most recently, Ford (1994), using a one-layer model (which restricts applicability to the real atmosphere), determined that regions of zero or negative potential vorticity emitted much more intense gravity wave activity (including shocks) than cyclonic regions.

The generation of gravity waves by geostrophic adjustment and inertial instability could then lead to CAT when the gravity waves break. One possibility is that the waves would have phase speeds similar to the ambient upper-tropospheric flow (Fritts and Luo 1992) and would therefore encounter critical surfaces in the upper troposphere, depositing their pseudomomentum near regions of frequent aircraft travel (D. McCann 1995, personal communication).

In summary, inertial instability does occur in regions of strong anticyclonic flow, and weakly inertially stable or inertially unstable conditions should be accompanied by gravity wave generation. It also seems plausible that the most intense gravity wave activity should coincide with regions of inertial instability. This could lead to CAT generation in regions that exhibit relatively low values of flow deformation (see section 2). Unfortunately, current theory and modeling work do not permit a more detailed understanding of the process of gravity wave generation by inertial instability.

5. Relevance to conventional CAT diagnostics

The discussion in section 4 suggests that it is possible that strongly anticyclonic regions can generate CAT through means that are not accounted for in conventional CAT theory. While this certainly does not invalidate the existing understanding of CAT, it does refine our knowledge of CAT-producing mechanisms inherent in strong anticyclonic flow. Now we review some conventional CAT diagnostics in light of these findings.

None of the conventional CAT diagnostics—Richardson number, SCATR index, or TI—incorporate the dynamical effects of strong anticyclonic flow discussed in sections 3 and 4. For example, diagnostics predicated entirely upon horizontal deformation, which incorporate neither vertical shear nor flow curvature, may underestimate CAT occurrences. In addition, none of the diagnostics take into account the inertial instability-imposed asymmetry between highs and lows, and thus the gravity wave generation by inertial instability. Examining each diagnostic, we find that some are less valid than others:

- The least valid CAT diagnostics are Brown's [1973, his Eq. (4)] and Dixon's (1976, unpublished manuscript) early versions of the SCATR index. Brown's form uses the gradient wind equation to derive the following (with some empirical tweaking):

$$\mathrm{SCATR}_b = [0.3(\zeta + f)^2 + \mathrm{DST}^2 + \mathrm{DSH}^2]^{1/2}, \quad (9)$$

and Dixon's is (Dutton 1980)

$$\mathrm{SCATR}_d = D^2 + (\zeta + f)^2 - \mathrm{DEF}^2. \quad (10)$$

These versions of SCATR clearly do not recognize any of the aforementioned differences between strongly anticyclonic and cyclonic flow. For example, in very anticyclonic flow the gradient wind approximation fails, and so does (9). Even though these forms of SCATR are not in common use, they are the only CAT indices in the literature that make explicit reference to the absolute vorticity, the natural diagnostic of inertial stability. As such, it must be emphasized that these diagnostics are not valid for strongly anticyclonic flows.

- Keller's (1990) SCATR index relates DST and DSH to tendencies of the Richardson number:

$$\mathrm{SCATR} = 2f\left|\frac{\partial \mathbf{v}_g}{\partial\theta}\right|\left|\frac{\partial \mathbf{v}}{\partial\theta}\right|^{-1}\cos\beta - \mathrm{DST}\cos2\alpha + \mathrm{DSH}\sin2\alpha, \quad (11)$$

where α is the clockwise angle between $\partial\mathbf{v}/\partial\theta$ and north, and β is the angle between $\partial\mathbf{v}/\partial\theta$ and ∇T. Here, the sin is of omission rather than commission: the process of gravity wave generation by inertial instability is poorly captured at best in SCATR—that is, only when Scorer's mechanism for inertial instability-related vertical shear production leads to strong tendencies in Ri. Regions of nearly barotropic strong anticyclonic shear are sometimes observed, which could lead to gravity wave production without large initial changes in Ri. Any CAT due to inertial instability generation of gravity waves would likely be poorly predicted.

- Ellrod and Knapp's (1992) TI are

$$\mathrm{TI1} = \frac{\Delta\mathbf{v}}{\Delta z}\mathrm{DEF}; \quad (12)$$

and

$$\mathrm{TI2} = \frac{\Delta\mathbf{v}}{\Delta z}(\mathrm{DEF} - D). \quad (13)$$

These forms are based on the Petterssen frontogenesis equation, which, as has already been argued, is not appropriate in and near centers of strong anticyclonic flow. Nevertheless, by (5), large negative values of ζ can be perceived as large values of DEF in nearly pure shear flows. The TI may thus erroneously interpret strongly anticyclonically sheared regions as frontmakers, and then for the wrong reasons may sometimes predict CAT correctly. However, for sharply sheared *and* curved ridges, the TI may underpredict CAT because it does not incorporate geostrophic adjustment and inertial instability into the calculation.

Given that most or all existing CAT indices are flawed in regions of strongly anticyclonic flow, a question arises: what would be a better predictor of turbulence in these regions? Classical theory would suggest some function of absolute or potential vorticity, based on (8). Arakawa (1952) anticipated turbulence for both strong anticyclonic and cyclonic shear; Schaefer and Hubert (1955) tentatively suggested a composite of absolute vorticity and Richardson number as a predictor of CAT. Previous observational studies (e.g., Dutton 1980) have sought in vain for a strong linear correlation between absolute vorticity and CAT. According to the arguments presented here, however, a *nonlinear* relationship might exist instead: a maximum of CAT at $\zeta + f \approx 0$ in addition to the well-known maximum when $\zeta + f \gg 0$ with a minimum in-between, as suggested in Sparks et al. (1977, their Fig. 5.4). This relationship may often be masked, however, by the relative rarity of strong anticyclonic flow events. In any event, application of an absolute vorticity-based CAT diagnostic would be premature, since the connection between absolute vorticity and gravity wave generation is poorly quantified at present. Finally, a deeper question concerns how relevant classical inertial instability theory is to observed flow regimes; this point is being addressed in ongoing research (Knox 1996). Clearly, more research is required on this subject.

6. Summary

Our understanding of clear-air turbulence is not yet complete, despite decades of research. One line of research that has languished is the relationship between CAT and strongly anticyclonic flows. Although rarer than its cyclonic counterpart, strongly anticyclonic flow could possibly account for a significant fraction of the CAT events that escape prediction. Simple arguments have been advanced here to explain why conventional CAT measures may be inadequate in regions of strong anticyclonic flow. Furthermore, work done in middle-atmosphere dynamics and fluid dynamics seems to offer mechanisms for gravity wave generation in strongly anticyclonic flows: geostrophic adjustment and inertial instability, processes that are not accounted for in current methods of predicting and diagnosing CAT.

The next step is to definitively link a robust case of geostrophic adjustment and/or inertial instability with a series of moderate and severe CAT reports. Ongoing research at the Aviation Weather Center is devoted to this subject (D. McCann 1996, personal communication). Given this connection, the aviation forecasting community will require a much better understanding than is currently available concerning the characteristics of gravity wave generation from weakly inertially stable and inertially unstable regions. It is hoped that this discussion will stimulate theoretical and modeling interest in this direction.

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